Problem 24

Grains of fine California beach sand are approximately spheres with an average radius of 50 μm and are made of silicon dioxide, which has a density of 2600 kg/m^3. What mass of sand grains would have a total surface area (the total area of all the individual spheres) equal to the surface area of a cube 1.00 m on an edge?

Solution

Multiply the density by the volume of a sphere to get the mass of a single grain of sand.

$$m = \rho V = \frac{2600 \text{ kg}}{1 \cancel{\text{m}^3}} \times \frac{4}{3}\pi \left(50 \cancel{\mu\text{m}} \times \frac{1 \cancel{\text{m}}}{10^6 \cancel{\mu\text{m}}}\right)^3 \approx 1 \times 10^{-9} \text{ kg}$$

Divide this mass by the surface area of a sphere to get the mass per unit surface area.

$$\frac{m}{A} = \frac{\rho V}{4\pi r^2} = \frac{\frac{2600 \text{ kg}}{1 \cancel{\text{m}^3}} \times \frac{4}{3}\pi \left(50 \cancel{\mu\text{m}} \times \frac{1 \cancel{\text{m}}}{10^6 \cancel{\mu\text{m}}}\right)^3}{4\pi \left(50 \cancel{\mu\text{m}} \times \frac{1 \text{ m}}{10^6 \cancel{\mu\text{m}}}\right)^2} = \frac{13}{300} \frac{\text{kg}}{\text{m}^2}$$

A cube with edge length 1.00 m has a total surface area of $6 \times (1.00 \text{ m})(1.00 \text{ m}) = 6.00 \text{ m}^2$ due to having six faces. Therefore, the mass of sand with this surface area is

$$6.00 \cancel{\text{m}^2} \times \frac{13}{300} \frac{\text{kg}}{\cancel{\text{m}^2}} = 0.260 \text{ kg}.$$